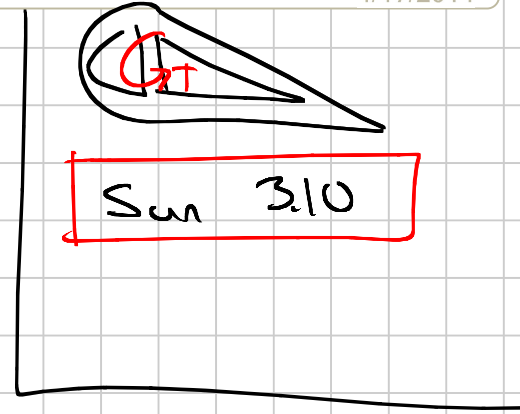
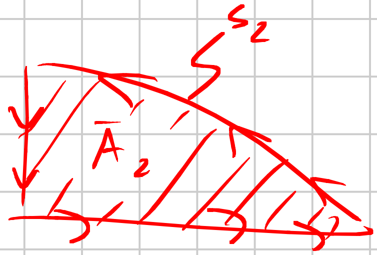
Aerospace Structures 19

Note Title 4/17/2014

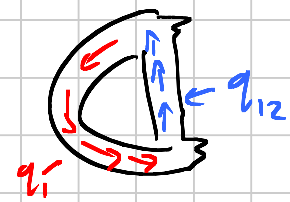


$$\bar{A}_1 = \frac{\pi(.6)^2}{2} = .565\,m^2$$



$$\bar{A}_2 = \frac{1}{2}(2)(1.2) = 1.2\,m^2$$

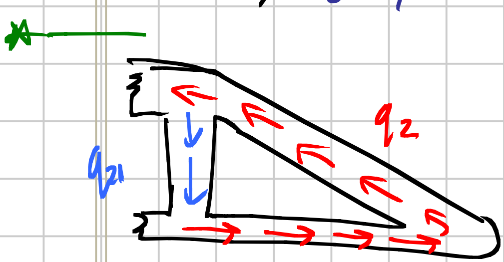
$$T = \sum_K 2\bar{A}_K q_K = 2(.565)q_1 + 2(1.2)q_2$$



$\Rightarrow 2\times10^5\ N\cdot m = 1.13 q_1 + 2.4 q_2$ — (1)

$$\theta_1 = \frac{1}{2\bar{A}_1 G}\oint \frac{q}{t}\,ds = \frac{1}{2\bar{A}_1 G}\left[\frac{q_1(\pi R)}{t_1} + \frac{\overbrace{q_{12}}^{q_1 - q_2}(2R)}{t_{12}}\right]$$

$\Rightarrow G\theta_1 = 531 q_1 - 212 q_2$ — (2a)



$$\theta_2 = \frac{1}{2\bar{A}_2 G}\oint \frac{q}{t}\,ds + \frac{1}{2\bar{A}_2 G}\left[\frac{q_2(2)}{t_2} + \frac{q_2\sqrt{2^2+1.2^2}}{t_2} + \frac{\overbrace{q_{21}}^{q_2-q_1}(1.2)}{t_{21}}\right]$$

$$= \frac{1}{2(1.2)G}\left[\frac{q_2(2)}{.007} + \frac{q_2(2.33)}{.007} + \frac{(q_{21})(1.2)}{.005}\right]$$

$\Rightarrow G\theta_2 = 358 q_2 - 100 q_1$ — (2b)

$\boxed{\theta_1 = \theta_2}$ "compatability relation"

$G\theta_1 = G\theta_2 \rightarrow 531 q_1 - 212 q_2 = 358 q_2 - 100 q_1$

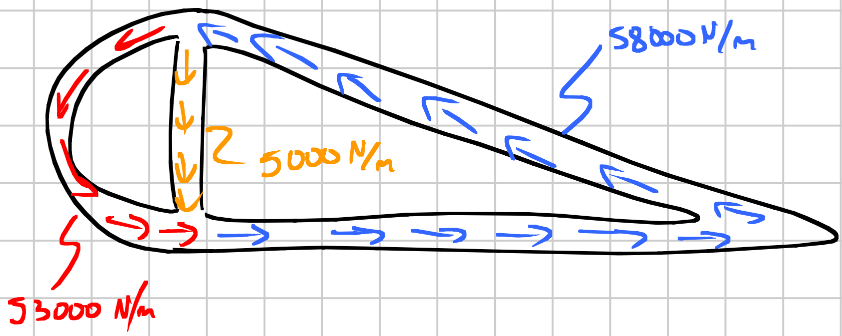
$q_2 = 1.1 q_1$ — (3)

(3 into 1)

$$2\times10^5 = 1.13q_1 + 2.4(1.1q_1)$$

$$\Rightarrow q_1 = 53\,\text{kN/m} = 5.3\times10^4\,\text{N/m} \quad \text{(4a)}$$

$$\Rightarrow q_2 = 58000\,\text{N/m} = 5.8\times10^4\,\text{N/m} \quad \text{(4b)}$$



Shear Stress

$$T_1 = \frac{q_1}{t_1} = \frac{53000}{.003} = 10.6\times10^6\,\text{N/m}$$

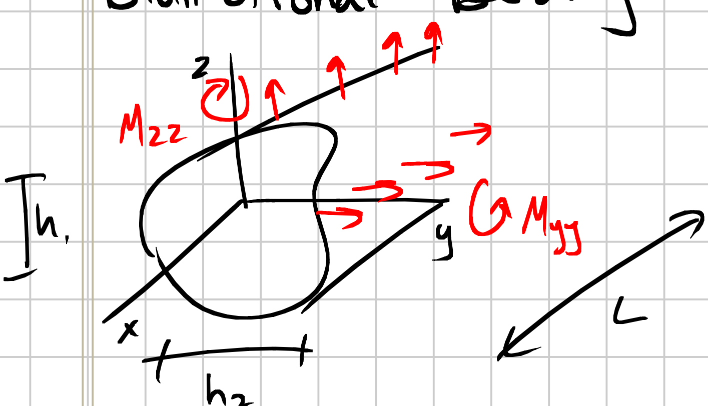
$$T_2 = \frac{q_2}{t_2} = \frac{58000}{.007} = 8.29\times10^6\,\text{N/m}$$

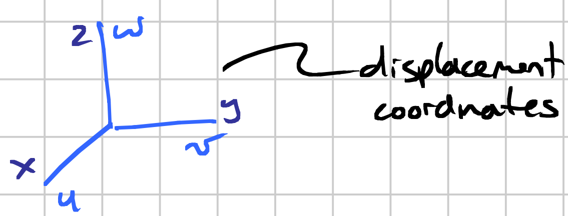
$$T_3 = \frac{q_{21}}{t_{12}} = \frac{5000}{.0005} = 1\times10^6\,\text{N/m}$$

Bidirectional Bending

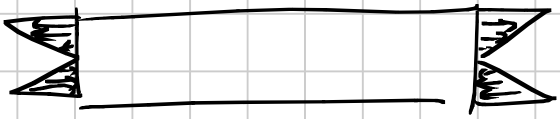
→ long, thin structures

$h_1, h_2 \ll L$

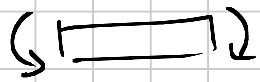




→ Recall from Stress/Strain



"pure bending"



Kirchhoff Assumption: rigidity in "thickness" directions

$$\left.\begin{aligned} x &\rightarrow u(x,y,z) = u_0(x) + z\,\psi_y(x) + y\,\psi_z(x) \\ y &\rightarrow v(x,y,z) \approx v_0(x) \\ z &\rightarrow w(x,y,z) \approx w_0(x) \end{aligned}\right\} \text{—(1)}$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial x} + \underbrace{\left[\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]}_{\text{Geometric Non-linearities}} \quad \text{—(2)}$$

(1) into (2)

$$\varepsilon_{xx}(x,y,z) = \varepsilon_0(x) - y\,k_{zz} - z\,k_{yy} \quad \text{—(2')}$$

$$K_{zz} = \frac{\partial \psi_z}{\partial x} \simeq^{*} -\frac{\partial^2 w_0}{\partial x^2}, \quad K_{yy} = \frac{\partial \psi_y}{\partial x} \simeq^{*} -\frac{\partial^2 v_0}{\partial x^2} \quad (K_{zz}, K_{yy}: \text{Curvature Changes}) \quad \text{—(3a)}$$

$* \tan(\psi) \simeq \psi, \quad \psi \ll 1$

Hooke's Law

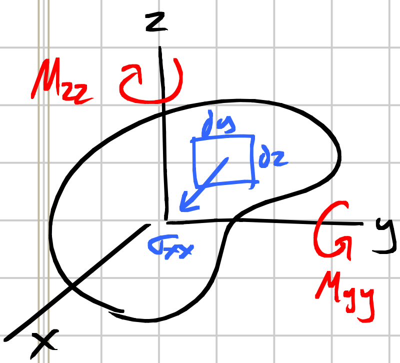
$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}\left(\sigma_{yy} + \sigma_{zz}\right) \quad \text{—(4)}$$

(assume $\sigma_{yy}, \sigma_{zz} \ll \sigma_{xx}$)

$$\Rightarrow \sigma_{xx} \simeq E\,\varepsilon_{xx} \quad \text{—(4')}$$

(2) into (4')

$$\Rightarrow \sigma_{xx}(x,y,z) = \underbrace{E\,\varepsilon_0(x)}_{\sigma_{\text{Membrane}}} - \underbrace{\left[E y \overbrace{\frac{\partial^2 v_0}{\partial x^2}}^{k_{zz}} + E z \overbrace{\frac{\partial^2 w_0}{\partial x^2}}^{k_{yy}}\right]}_{\sigma_{\text{Bending}}} \quad \text{—(5)}$$



$$M_{yy} = \int_A z\sigma_{xx}\, dA$$

$$M_{zz} = \int_A \sigma_{xx}\, dA$$

(3) into above

$$M_{yy} = -EI_{zy}k_{zz} - EI_{yy}k_{yy} \quad \text{— (6a)}$$

$$M_{zz} = -EI_{zz}k_{zz} - EI_{yz}k_{yy} \quad \text{— (6b)}$$

Where:

$$I_{zz} = \int_A y^2\, dA \quad \text{— (6c)}, \quad I_{zz} = \int_A z^2\, dA \quad \text{— (6d)} \quad (2^{nd} \text{ Moments of Inertia})$$

$$I_{zy} = I_{yz} = \int_A zy\, dA \quad \text{— (6e)} \quad (\text{Products of Inertia})$$

If y on z axis is axis of symmetry,

$$\underline{I_{zy} = I_{yz} = 0}$$